

# Module V: Engineering Analysis: Introduction to finite element method; Principle of potential

energy; FE analysis of 1D element problems (spring, bar, truss elements); Development of element stiffness equation and their assembly; Plain strain and plain stress problems; Domain discretization, pre-processing and post-processing; Verification and validation; Popular CAE software used in industry

Certainly! Below is a detailed breakdown of:

## Module V: Engineering Analysis

### *Computer Aided Design & Analysis*

This module introduces students to the **Finite Element Method (FEM)**, a computational tool widely used for **engineering analysis of structures, components, and systems**. It also covers the application of FEM in 1D element problems (springs, bars, trusses) and the principles behind stress-strain analysis. Students are also exposed to the usage of **Computer-Aided Engineering (CAE)** tools for real-world simulation and analysis.

### 1. Introduction to the Finite Element Method (FEM/FES)

#### ▮ What is FEM?

- A **numerical technique** used to solve differential equations and approximate solutions for complex physical systems.
- Divides the domain into **finite smaller elements**, solves for each element, and assembles the global solution.

#### ▮ Applications:

- Structural analysis
- Thermal analysis
- Dynamic and vibration analysis
- Fluid flow (CFD using FVM/FE)
- Electromagnetic field simulation

## 2. Principle of Potential Energy (PPE)

### ▮ Statement:

A system in equilibrium will have a minimum total potential energy.

### ▮ Use in FEM:

Used to derive element stiffness equations:

- **Total Potential Energy = Strain Energy – Work Done by Applied Loads**
- The variation ( $\delta$ ) of this total potential energy leads to the system of FEM equations:

$$\delta\Pi = 0 \Rightarrow Ku = F$$

## 3. Finite Element Analysis of 1D Element Problems

Basic formulation based on displacement-based approach:

### a) Spring Element:

#### • Governing equation:

$$k_e = \frac{EA}{L}$$

\$\$

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

\cdot

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

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**b) Bar Element:**

- Axial deformation under loading:
  - Element stiffness matrix is derived from virtual work or potential energy principle.

**c) Truss Element:**

- Has axial forces only; used for pin-jointed structures.
- Element stiffness matrix derived considering orientation angle ( $\cos\theta$ ,  $\sin\theta$  terms).

**4. Development of Element Stiffness Equation and Assembly****a) Element Stiffness Matrix:**

Represents how an element resists deformation per applied force/displacement.

**b) Global Stiffness Matrix Assembly:**

- Assemble all element matrices into a global system:
$$[K_{global}]\{u\} = \{F\}$$
- Apply **boundary conditions** to simplify and solve the system of equations.

**5. Plane Stress and Plane Strain Problems****a) Plane Stress:**

- For thin plates subjected to in-plane loads ( $\sigma_z \approx 0$ ).
- Common in sheet metal components like plates, brackets.

**b) Plane Strain:**

- For long bodies where deformation in one direction (usually out-of-plane) is negligible ( $\epsilon_z \approx 0$ ).
- Common in dams, earth structures, etc.

**Governing Equations include:**

- **Constitutive (Stress-Strain):**
  - Elastic matrix **[D]** differs for plane stress and strain.
- **Element shapes:**
  - 3-node triangle (CST), 4-node quadrilateral (Q4), 8-node (Q8) elements.

6. Domain Discretization, Pre-processing & Post-processing

a) Domain Discretization:

- Dividing the entire physical model into smaller parts or elements (mesh).
- Types: 1D (line), 2D (triangular, quadrilateral), 3D (tetrahedral, hexahedral).

b) Pre-processing:

- Geometry definition and import
- Material property assignment
- Boundary conditions (constraints and loads)
- Mesh generation

c) Post-processing:

- Visualization of results:
  - Displacements
  - Stress and strain distributions
  - Contour plots, deformed shapes, animations
- Interpretation, validation, and reporting

7. Verification and Validation (V&V)

Stage	Definition
Verification	Ensures the computational model is solved correctly (e.g., mesh independence study).
Validation	Ensures the model solves the correct real-world physical problem (experimental comparison).

- Necessary to ensure **accuracy, reliability, and acceptance** of CAE results in engineering design or certification.

8. Popular CAE Software in Industry

Software	Application Domains
ANSYS	Multiphysics, structural, thermal, fluids
Abaqus	Advanced non-linear FEA, materials, impact
SolidWorks Simulation	Integrated with CAD (SolidWorks)
COMSOL Multiphysics	Coupled physics modeling
HyperWorks (Altair)	Optimization, crash analysis, meshing
Siemens NX CAE	High-end integrated CAD/CAE

## ✓ Summary Table

Concept	Description
FEM	Numerical method for solving physical systems
PPE	Basis for forming stiffness matrices
1D Element Models	Spring, bar, truss with direct stiffness method
Plane Problems	Stress (thin), Strain (long) approximations
Discretization	Domain meshing — critical to analysis accuracy
V&V	Ensures correctness and applicability
CAE Tools	Software for simulation and analysis

## □ Conclusion

This module provides foundational knowledge in **Finite Element Analysis (FEA)** relevant for structural and mechanical systems. Mastery of these techniques enables engineers to:

- Analyze stresses and displacements,
- Optimize product performance,
- Validate design decisions,
- Reduce physical prototyping time via simulation.

Let me know if you'd like:

- Example FEA problems with solutions,
- Matrix assembly demonstrations, or
- Software lab activity templates using tools like ANSYS or Abaqus.